PHYSICAL REVIEW E VOLUME 62, NUMBER 3 SEPTEMBER 2000

Suppressing complexity via the slaving principle

Shunguang Wu,[1,2,3,4,*] Kaifen He,[1,2,3,4] and Zuqia Huang[1,2,3]
[1]*Key Laboratory in University for Radiation Beam Technology and Materials Modification, Beijing Normal University, Beijing 100875, People's Republic of China*
[2]*Institute of Low Energy Nuclear Physics, Beijing Normal University, Beijing 100875, People's Republic of China*[†]
[3]*Beijing Radiation Center, Beijing 100875, People's Republic of China*
[4]*CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, People's Republic of China*
(Received 4 January 2000; revised manuscript received 2 June 2000)

The complexity of a nonlinear dynamic system can be suppressed by adding an external period force with appropriate choice of frequency and amplitude directly on the slowly changing variable of the system. Numerical results indicate that the method not only can suppress chaos but also is robust to the additive external noise.

PACS number(s): 05.45.−a, 52.35.Kt, 52.35.−g

In many nonlinear dynamic systems, the dynamic variables can be divided into the fast changing variable(s)(FVs) and the slow changing variable(s)(SVs). The average periods of the FVs are relatively shorter than those of SVs. On the other hand, the slaving principle [1] tells us that the FVs can be slaved by the SVs. Hence, one can conceive that, if we can tame the SVs in a nonlinear dynamical system by some method, the FVs will be transited into simple motions from the complexity ones by the slaving principle. Thus the complexity of a dynamical system can be suppressed.

Enlightened by the above idea, we suggest a method of suppression complexity through the following routes: (i) search for an SV and its maximum amplitude, I_0, in the nonlinear dynamic system by time series analysis; (ii) mark the main oscillation frequency region, $[\omega_1,\omega_2]$, of the SV via its power spectrum density analysis; (iii) add an external force, $\eta\cos(\omega t)$, directly to the SV; (iv) search for the suitable parameters η and ω in the range $0-0.1I_0$ and $[\omega_1,\omega_2]$ respectively, with which the complexity of the system can be controlled. The controlling results can be convinced by observing the time series or calculating the Lyapunov exponent spectra (LES) of the system.

In fact, the above controlling method is the so-called ''open-loop'' scheme [2]. In contrast to the ''closed-loop'' technique inspired by Ott *et al.* [3], which needs a fast responding feedback system to produce an external signal in response to the system's dynamics, the algorithms of ''open loop'' scheme are independent of the system state, e.g., it does not need feedback response system. Although using the ''open-loop'' methods on the system to control complexity have been studied extensively in the past decade [4–8], within our knowledge, most of them are carried out by parameter perturbation, only a few study the controlling mechanisms and add the external force directly to the system. In this paper, we present examples of controlling complexity by using the slaving principle, and discuss the effects of additive noise on the controlled system. The effectiveness of the method is tested by numerical simulations. We expect that it could be realized in experiments.

This paper is organized as follows. First, we give three examples to demonstrate the effectiveness of the controlling scheme. Then, the mechanism of the controlling method is analyzed, which is attributed to the slaving principle. Next, the effects of additive noise on these systems are discussed. A brief conclusion and potential applications of the method are given lastly.

The first example is the Rose-Hindmarsh (RH) model, which can be used to describe the bursting behavior of neurons [9]. It reads

$$\begin{aligned}\dot{x}&=y-ax^3+bx^2+I-z,\\ \dot{y}&=c-dx^2-y,\\ \dot{z}&=r[s(x-x^*)-z],\end{aligned}\tag{1}$$

where x is the electrical potential of the biology membrane, y is the recovering variable, z is the adjusting current, the parameters are set at the standard values $a=1$, $b=3$, $c=1$, $d=5$, $x^*=1.6$, $s=4$, and only r and I are chosen as the control parameters. The LES of the system at $(I=2.8,r=0.013)$ are $(0.2,0,-1)$ and the corresponding chaotic trajectory is shown in Fig. 1(a).

Comparing the time series of the three variables, one can find that z is the SV. Hence, we add an external force $\eta_1\cos(2\pi f_1 t)$ directly on the right-hand side of the third equation, and hope that the chaos can be controlled by choosing parameters η_1 and f_1 properly. To this end, the following three steps are considered: (i) limit the candidate frequencies in the range $[5,25]$ by analyzing the power spectrum densities of variable z [Fig. 1(b)]; (ii) confine the proper η_1's in the range $[0,0.2]$ according to the amplitude of variable z; (iii) select the suitable parameters via computing the LES. As a result, the LES of the system under control are displayed in Fig. 1(d). Figure 1(c) shows the controlled trajectory of system (1) at $(\eta_1=0.02,f_1=10)$, the corresponding power spectrum densities S_2 of variable z are shown in Fig. 1(b). It is clear that the chaos has been suppressed.

*Corresponding author. Email address: szzhang@bnu.edu.cn
[†]Mailing address.

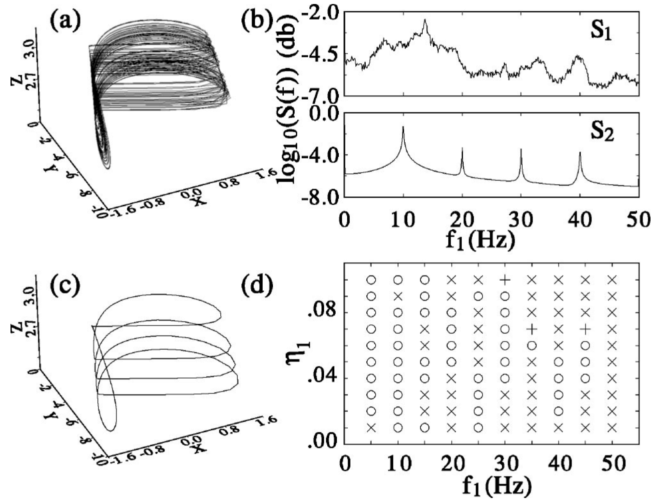

FIG. 1. (a), (c) The trajectories of system (1) before and after the controlling with control parameter $\eta_1=0.02, f_1=10$. (b) The power spectrum densities of variable z before (S_1) and after (S_2) the controlling. (d) The LES of system (1) on the control parameter plane, where the circles, pluses and crosses stand for three cases of LES as $(0,-,-)$, $(0,0,-)$, and $(+,0,-)$, respectively.

The second example is the Pikovsky circuit, which is an autostochastic generator. It can be expressed as [10]

$$\dot{x}=y-\delta z,$$

$$\dot{y}=-x+2\gamma y+\alpha z+\beta, \tag{2}$$

$$\mu\dot{z}=x-z^3+z,$$

where x,y,z stand for the characterized quantum of the circuit, δ, γ, β, μ, and α are parameters that can be determined by the elements of the circuit. In this study, we take $\beta=0$, $\delta=0.66$, $\alpha=0.165$, $\gamma=0.17$, and choose μ as the control parameter. Numerical simulation results show that system (2) displays a chaotic motion when $\mu=0.047$. To suppress chaos, we add an external force $\eta_2\cos(2\pi f_2 t)$ on the right-hand side of the third equation in system (2), and use the methods to search for the suitable control parameters. It indicates that there indeed exist some suitable parameters. As an example, the trajectories of system (2) before and after control are shown in Figs. 2(a) and 2(b), respectively.

The third example is an injection two-level laser system [11]. It reads

$$\dot{E}_1=\beta(\theta E_2-E_1)+\beta F+2\beta(P_1E_1+P_2E_2),$$

$$\dot{E}_2=-\beta(E_2+\theta E_1)+2\beta(P_2E_1-P_1E_2),$$

$$\dot{P}_1=-(P_1-\Delta P_2)+D(E_1^2-E_2^2), \tag{3}$$

$$\dot{P}_2=-(P_2+\Delta P_1)+2DE_1E_2,$$

$$\dot{D}=-\sigma(D-R)-4(P_1E_1^2-P_1E_2^2+2P_2E_1E_2),$$

where $E(=E_1+iE_2)$ is the intensity of the laser field, P $(=P_1+iP_2)$ is the polarization intensity of the media, D is the density of population inversion, F stands for the intensity

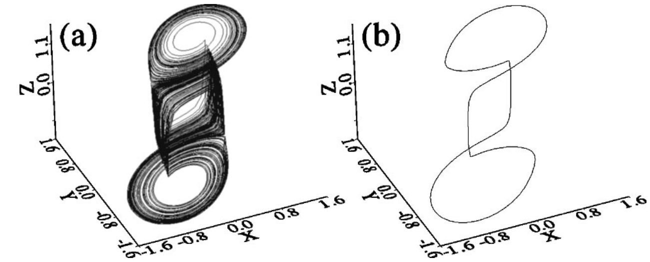

FIG. 2. The trajectories of system (2) before (a) and after (b) the controlling with control parameters at $(\eta_2=0.288, f_2=0.2)$.

of the external injection field, and β, σ, θ, Δ, and R are the system parameters. The parameters are set at the standard values [11] $\beta=0.45$, $\sigma=0.04$, $\theta=150$, $\Delta=100$, and $F=150$, and only R is chosen as the control parameter. Numerical studies indicate that when $R=350$, system (3) shows a chaotic motion. By time series analysis, one can find that E_1 or E_2 are slow variables, hence, we add an external force $\eta_3\cos(2\pi f_3 t)$ on the right-hand side of the first equation in system (3) to control the chaos. Suitable control parameters can be found by this method. As an example, Figs. 3(a) and 3(b) show part of the trajectory on E_1-P_1 plane before and after control.

To analyze the mechanism of the control method mentioned above, let us observe the transient time of variables from a chaotic state to a periodic state when a control signal is added to the three systems. For describing it clearly, we take the RH model as an example. From the time series of x, y and z in system (1), one can see that the transient time, $\tau_k^*(k=x,y,z)$, of the SV and FVs are different. In order to determine τ_k^*, let us calculate the relative average amplitude value of each variable as

$$r_k(t)=|v_k(t)/\overline{v_k}| \quad (k=x,y,z), \tag{4}$$

where $\overline{v_k}$ is the time average value of v_k in the periodic state and v_k's are the points on the Poincaré section [12]. For example, the results of $r_x(t)$, $r_y(t)$, and $r_z(t)$ are shown in Fig. 4, and one can see that, after adding the control signal, $r_k(t)$ oscillates around 1 in the early stage, and approaches to 1 asymptotically. In order to analyze this process in more detail, we define the relative average errors of each variable as

$$\epsilon_k(t)=|v_k(t)-\overline{v_k}|/\overline{v_k} \quad (k=x,y,z). \tag{5}$$

The inset in Fig. 4 displays the corresponding results for the RH model. It shows that in the decreasing phase of ϵ_k, it

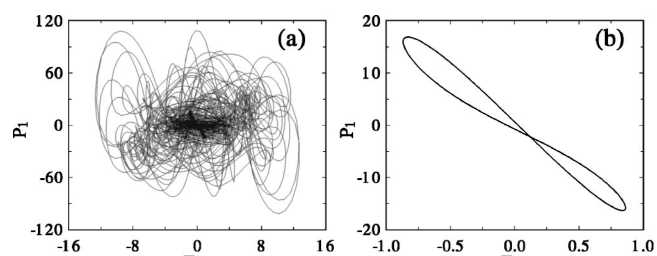

FIG. 3. The trajectory of system (3) before (a) and after (b) the controlling with control parameters at $(\eta_3=0.96, f_3=12.5)$.

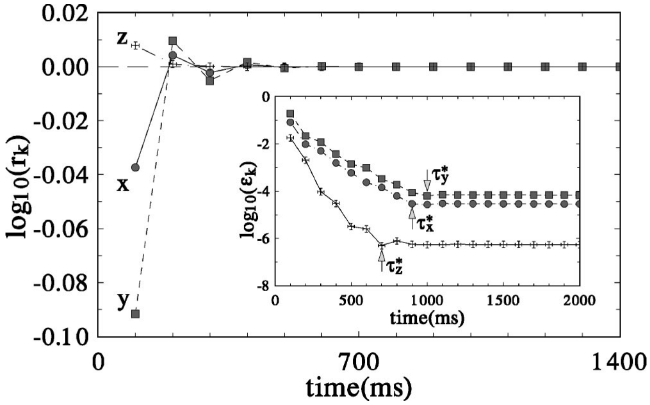

FIG. 4. The variables' relative amplitude value r_k and their relative average errors ϵ_k(the inset) as a function of time for the RH model.

almost decreases linearly in the logarithmic diagram, while in the periodic phase, ϵ_k becomes almost a constant. The turning point gives τ_k^*.

Using the same trick mentioned above, we obtain τ_k^*'s for the Pikovsky circuit and the laser system. The results of the three systems are shown in Fig. 5. It depicts that the SV mode is controlled firstly, then the faster ones.

Moreover, to test if the results, obtained in Fig. 5, are dependent on the modes to which the control signal is added, we add the control signal to FVs to control chaos. The typical results are shown in Table I. For the RH model, if we add control signal, $0.07\cos(80\pi t)$, to the fastest mode y (*the fast mode controlling*), the transient times are $\tau_x^*=3500$, $\tau_y^*=3800$, $\tau_z^*=1400$; while if we add $0.09\cos(50\pi t)$ to another fast mode x (*the middle mode controlling*), the results are $\tau_x^*=1120$, $\tau_y^*=1280$, $\tau_z^*=960$. Comparing with the *slow mode controlling* results, $\tau_x^*=900$, $\tau_y^*=1000$, $\tau_z^*=700$ [Fig. 5(a)], we find that (i) in any case, the SV is controlled firstly, the fast one later, and the fastest one the last; and (ii) the transient times for the slow mode controlling are the shortest ones, for the fast mode controlling is the longest one. One can see the similar results in the Pikovsky circuit. For the laser system, however, we cannot find out suitable control parameters to suppress chaos in the studied parameter regime, if the control signal is added to the FVs.

From these results we conclude that, by using our time-periodic signal to control chaos, the variables will be controlled one by one from SV to FVs. The faster the variable to which the signal is added, the longer transient time is needed if the system can nevertheless be controlled. This phenomenon can be understood by the slaving principle in synergetics [1]. In the transient stage, an SV, which is affected by the control force, plays an important role as an order parameter in the system, and the FVs can be slaved by the SV. Because of different nonlinear coupling intensities, the transient time of FVs are different.

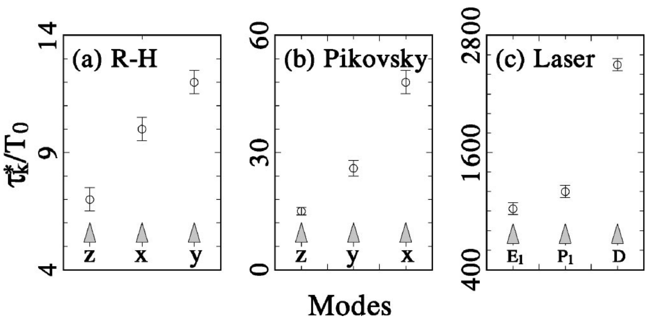

FIG. 5. The transient time, τ_k^*, as a function of system variables for the three systems. Here T_0 stands for the period of the controlling signal, it equals 0.1s, 5s, and 0.08s for (a), (b), and (c), respectively.

TABLE I. The transient time for different models while the controlling signal is added to the FVs.

Models	Signal added mode	(η, f)	$(\tau_x^*, \tau_y^*, \tau_z^*)$
RH	x	(0.090, 25.0)	(1120, 1280, 960) ms
	y	(0.070, 40.0)	(3500, 3800, 1400) ms
Pikovsky	x	(0.256, 0.50)	(400, 470, 355) s
	y	(0.032, 0.25)	(64 , 76, 35) s

Now, we discuss if the aforementioned controlling technique is robust to an external noise. To this end, we add an additive noise directly to the three systems. Hence, the relative equations of systems (1), (2), and (3) can be written, respectively, as

$$\dot{z}=r[s(x-x^*)-z]+\eta_1\cos(2\pi f_1 t)+\sqrt{2D_1}\xi(t), \quad (6)$$

$$\mu\dot{z}=x-f(z)+\eta_2\cos(2\pi f_2 t)+\sqrt{2D_2}\xi(t), \quad (7)$$

and

$$\dot{E}_1=\beta(\theta E_2-E_1)+\beta F+2\beta(P_1E_1+P_2E_2)$$
$$+\eta_3\cos(2\pi f_3 t)+\sqrt{2D_3}\xi(t), \quad (8)$$

where $\xi(t)$ is a Gaussian white noise with the statistical properties $\langle\xi(t)\rangle=0,\langle\xi(t)\xi(t')\rangle=\delta(t-t'),D_i(i=1,2,3)$ is the noise intensity.

Mannella's algorithm [13] is used to numerically simulate the stochastic differential equations. To observe the effects of noise on the three systems, we define a quantity ρ to characterize it, which is expressed as

$$\rho_i=\left(\frac{h_i}{\Delta w_i}\right)\Bigg/\left(\frac{h_i^{(0)}}{\Delta w_i^{(0)}}\right) \quad (i=1,2,3), \quad (9)$$

where h_i is the peak height of the spectrum at frequency f_i, with noise intensity $D_i, \Delta w_i$ is the width of the peak at a height $h'=\frac{1}{2}h_i$, $h_i^{(0)}$, and $\Delta w_i^{(0)}$ stand for the peak height and width in the case without noise, respectively.

In the numerical simulation, the noise intensities are chosen from 1% to 50% of the amplitudes of the external forces. Figure 6 shows the results of ρ vs. relative noise intensities $R_d[R_d=\sqrt{2D_i}/\eta_i\ (i=1,2,3)]$ for the three systems. It can be seen that the RH model is the most robust one in the three systems, its spectrum density peak almost does not change even though the external noise intensity reaches 50% of the control signal's amplitude. The laser system is sensitive to the external noise very much, if the intensity of the noise

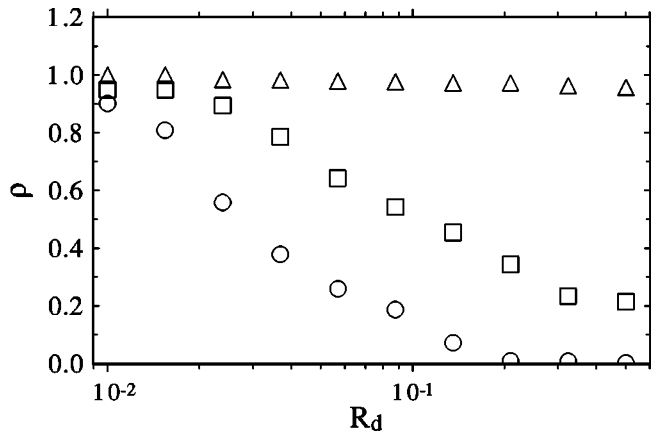

FIG. 6. ρ as a function of relative noise intensities, R_d, for the three systems. The triangles, squares, and circles are for systems (1), (2), and (3), respectively.

reaches 10% of the control signal's amplitude, ρ decreases to 0.2. The Pikovsky circuit is the intermediate one in the three systems.

In conclusion, we have successfully suppressed chaos by directly adding a periodic force on the system. Appropriate input frequencies can be chosen by analyzing the spectrum of the chaotic attractors. In the controlling process, the variables are controlled one by one from SV to FVs. The mechanism of the control method can be understood by the slaving principle. The effects of additive noise on the three typical systems show that the method is robust to environmental noise. In contrast to the feedback method for which one needs the knowledge of the target state, the present method is easier to be realized in practical experiments.

We argue that the method may have some potential applications on neuron science and laser technology. For example, one can use this idea to design a laser system with which the output frequency can be adjusted by the frequency of the external control signal, or select an ideal output frequency to improve the laser's quality. While for neurons, since the variables in some important neuron models [14] can be divided into SV and FVs, and the final controlled state is strongly dependent on the amplitude and frequency of the external signal (not shown in this report), the method may be used to study the rhythms of the neurons, and patterns of neuronal networks by the experimental biologists.

This work was supported by the Visiting Scholar Foundation of Key Laboratory in University of Chinese Education Department, the Post Doctoral Foundation of China, the RFDP under Grant No. 98002713, and the NSF of China under Grant No. 19975006, 19835020. The first author (S.W.) would like to thank Professor Gang Hu, Dr. Changjun Zhu, and Dr. Wei Ren for stimulating discussions.

[1] H. Haken, *Synergetics, An Introduction*, 2nd ed. (Springer-Verlag, Berlin, 1978).

[2] J.L. Breeden, Phys. Lett. A **190**, 264 (1994).

[3] E. Ott, C. Grebogi, and J.A. York, Phys. Rev. Lett. **64**, 1196 (1990).

[4] V.V. Alekseev and A.Y. Loskutov, Dokl. Akad. Nauk (SSSR) **292**, 1346 (1987) [Sov. Phys. Dokl. **32**, 116 (1987)]; R. Lima and M. Pettini, Phys. Rev. A **41**, 726 (1989); A. Azevedo and S.M. Rezende, Phys. Rev. Lett. **66**, 1342 (1991); L. Fronzoni, M. Giocondo, and M. Pettini, Phys. Rev. A **43**, 6483 (1991); Y. Braiman and I. Goldhirsch, Phys. Rev. Lett. **66**, 2545 (1991); W.X. Ding, H.Q. She, W. Huang, and C.X. Yu, *ibid.* **72**, 96 (1994); M. Ciofini, R. Meucci, and F.T. Arecchi, Phys. Rev. E **52**, 94 (1995); Z.L. Qu, G. Hu, G.J. Yang, and G.R. Qin, Phys. Rev. Lett. **74**, 1736 (1994).

[5] P. Colet and Y. Braiman, Phys. Rev. E **53**, 200 (1996).

[6] A. Uchida, T. Sato, T. Ogawa, and F. Kannari, Phys. Rev. E **58**, 7249 (1998).

[7] K.A. Mirus and J.C. Sprott, Phys. Rev. E **59**, 5313 (1999).

[8] S. Wu, Keifen He and Z. Huang, Phys. Lett. A **260**, 345 (1999).

[9] J.L. Hindmarsh and R.M. Rose, Proc. R. Soc. London, Ser. B **221**, 87 (1984).

[10] A.S. Pikovsky and M.I. Rabinovich, Dokl. Akad. Nauk (SSSR) **239**, 301 (1978) [Sov. Phys. Dokl. **23**, 183 (1978)].

[11] J.Y. Zhang, S. Wu, and Z. Jan, Chin. J. of Quantum Electronics, **15**, 83 (1998).

[12] Here, the Poincaré section is the time at which the control signal has a maximum value.

[13] R. Mannella and V. Palleschi, Phys. Rev. A **40**, 3381 (1989).

[14] R. FitzHugh, Biophys. J. **1**, 445 (1961); R.E. Plant, J. Math. Biol. **11**, 15 (1981); T.R. Chay, Physica D **16**, 233 (1985).